

Supplementary Material – Part 1 (S1): Theoretical Background

SUMMARY

S1-1. Hydrological instability in the State of Mato Grosso (Brazil)	2
S1-2. Hydrological instability and effects on the hydraulic turbine	4
S1-3. Key performance indicators in hydropower plants	5
S1-4. Biomimetics and bio-inspiration	6
S1-5. Collaborative animals	8
S1-6. Two-Alternative Forced Choice (2AFC)	10
S1-7. The Herfindahl-Hirschman Index (HHI)	11
Reference.....	12

S1-1. Hydrological instability in the State of Mato Grosso (Brazil)

One indicator that can be used to study hydrological instability is the Rainfall Anomaly Index (RAI), developed by van Rooy [1]. The RAI indicator can be calculated as follows:

$$RAI = 3 (P_i - \bar{P}) / (\bar{E} - \bar{P}) \quad (\text{SMP1.1})$$

where P_i is the precipitation at time i ($\text{mm} \cdot \text{month}^{-1}$); $\bar{P}$ is the mean monthly precipitation of the historical series ($\text{mm} \cdot \text{month}^{-1}$); $\bar{E}$ is the average of the ten lowest monthly precipitation values from the historical series ($\text{mm} \cdot \text{month}^{-1}$).

Based on the values obtained, the rainfall regime was classified according to the RAI methodology presented in Table SMP1.1, which is an interpretation of the results.

Table S1.1. Rainfall classification according to the RAI as per [1].

Rainfall Anomaly Index (RAI)	Rainfall classification
≥ 3	Extremely wet
2.00 to 2.99	Very wet
1.00 to 1.99	Moderately wet
0.50 to 0.99	Slightly wet
-0.49 to 0.49	Near normal
-0.99 to -0.50	Slightly dry
-1.99 to -1.00	Moderately dry
-2.99 to -2.00	Very dry
≤ -3	Extremely dry

To assess the accumulated effects of dry and wet periods in the historical series, the accumulated RAI (RAIs) can be used as follows:

$$RAI_s = \sum_{i=1}^n RAI_i \quad (\text{SMP1.2})$$

where: i is the number of months studied in the historical series, $i = 1, 2, \dots, n$; n is the total number of months in the historical series ($n = 12 \cdot 34 = 408$).

A study of the RAI from 1988 to 2022 in the state of Mato Grosso, Brazil, detected temporal trends/changes in the precipitation pattern in the state [2]. The observation of the time series of RAI values presented in this study highlights a significant annual alternation between dry and wet periods, as shown by the RAI in Figure SMP1.1.

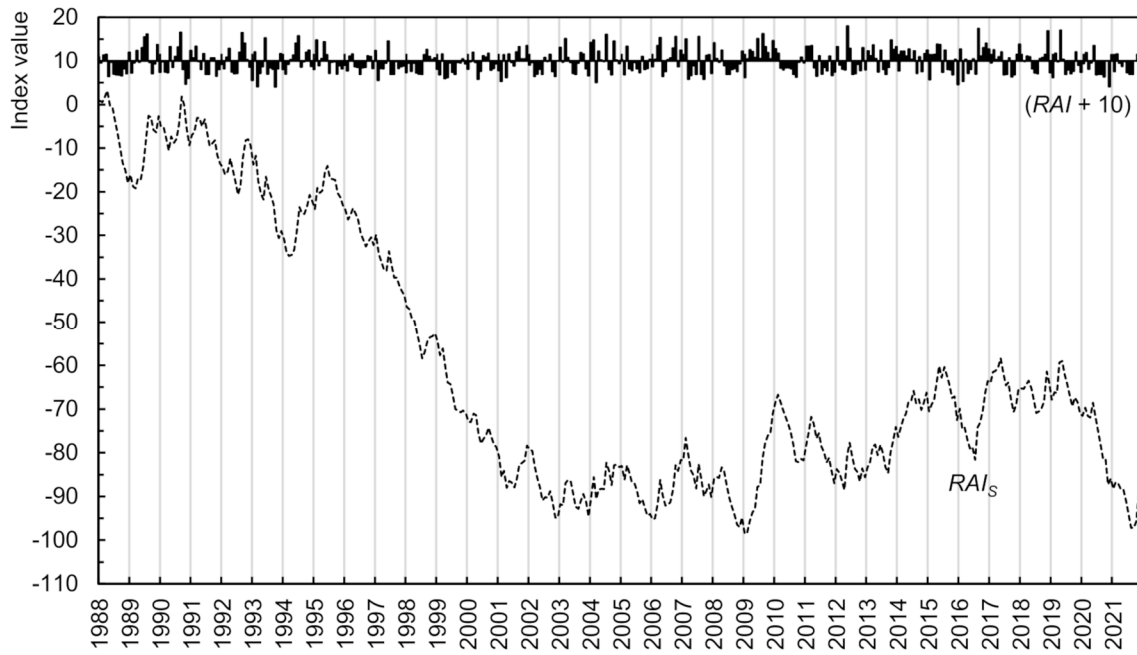


Figure S1.1. Time series of the Rainfall Anomaly Index indicators: RAI and RAI_s

The time series analysis of accumulated RAI values (RAI_s) suggests a trend of increasing intensity duration of dry periods, notably emphasising the period from 1996 to 2003, as shown by RAI_s in Fig.1. From 2003 to 2021, the alternation between dry and wet years can be observed, which serves as evidence of climatic instability, as shown by RAI_s in Fig.SMP1.1. Therefore, adaptation measures to hydrological instability, by local hydroelectric operators, can be considered important for achieving good operational performance.

S1-2. Hydrological instability and effects on the hydraulic turbine

Hydrological instability can render design and operational parameters (hydraulic head, inflows and outflows, among others) for a hydraulic turbine invalid, unknown, or unpredictable. Naturally, this relocates the turbine's operating point to a region different from the one envisaged in the design, commonly far from the operational optimum (lower efficiency, higher vibration and cavitation intensity, erosion wear, among others). The main effects of hydrological instability on a hydraulic turbine are presented below:

- Increased mechanical fatigue due to hydraulic transients and load fluctuations: irregular flow regimes induce rapid changes in discharge and pressure — causing water hammer and dynamic overloads in the penstock and rotating components, accelerating wear and reducing service life due to fatigue [3].
- Flow and hydraulic head variations reduce efficiency and cause operation outside the optimal range: during drought periods, lower flow rates and reduced head decrease output, but also lead to operational regimes outside the optimal range, potentially increasing vibrations and mechanical stress [4,5].
- Sediment erosion and cavitation during peak flows: intense rainfall transports abrasive sediments that wear blades and casings, causing surface degradation and cavitation, impairing performance and requiring shutdowns for maintenance [6].
- Flow instability and uneven distribution across blades and the rotor-stator system: unstable or asymmetric flow (hydraulic instability) can generate vibrations and adverse mechanical stresses in the rotor-generator system, accelerating structural deterioration [7].
- Downstream water level fluctuations (tailwater) during floods: oscillations in depth or pressure in the discharge water during start-up increase rotational instability, hindering smooth acceleration and causing dangerous oscillations [8].
- Thermal and electrical overloads in turbines and generators: off-design operation or repeated load/start-up cycles can generate electrical harmonics and torsional vibrations in the generator-turbine assembly, reducing component lifespan [9].
- Accelerated corrosion under extreme and variable operating regimes: alternating exposure to sediment-laden water, chemical variations, and oxidation promotes corrosion of metallic parts (blades, casings, valves), compromising structure and performance [10].

S1-3. Key performance indicators in hydropower plants

To enhance the safety and effectiveness of Operations and Maintenance (O&M) in hydroelectric plants, it is recommended that operators initiate the development of an O&M strategy with a comprehensive diagnosis of current performance. This diagnostic phase is essential for identifying gaps, setting priorities, and informing strategic decision-making [11]. The aim of the evaluation is to verify whether the safety metrics and measured performance meet internal and external goals, as well as industry best practice standards. The purpose of the diagnosis is to identify which aspects of O&M need to be improved and determine if the operator requires external assistance for the operation of their assets. Conducting the diagnosis is also a good preliminary tool to assess the condition of existing equipment, prioritise rehabilitation, and, if necessary, launch related feasibility studies to improve performance and ensure safe operation in the future [11]. The recommendation to prepare an O&M strategy is especially relevant for sites where hydrological instability is present. Hydrological instability puts pressure on the operational range of the hydroelectric plant and its performance indicators, resulting in the need for O&M programmes adapted to extreme conditions.

The traditional approach in the field of O&M focused on a single metric, reliability, to evaluate programs [12]. In general, O&M managers desire a reliable installation; however, this metric alone is not sufficient to assess or build a successful O&M's programme and strategy. In addition to reliability, O&M managers need to be responsible for controlling costs, evaluating and implementing new technologies, monitoring and reporting health, safety, and environmental issues, as well as expanding their programmes [12]. To support these activities, O&M managers must be aware of the various indicators that can be used to measure the quality or effectiveness of an O&M programme [12]. These metrics are useful not only for evaluating effectiveness but also for justifying costs in equipment acquisitions, programme modifications, and personnel hiring. Thus, adopting best practices is closely related to the best practice in key performance indicators (KPIs), and therefore, to the process of developing performance measures useful for an organization [13].

The KPI evaluation does not need to be exhaustive; instead, it can be a relatively high-level review that focuses on a list of the most relevant KPIs, thus generating customised KPIs for the case of interest. Further information on KPIs, as well as their appropriate selection, can be found in the following works [11–13].

S1-4. Biomimetics and bio-inspiration

Nature-inspired solutions have captivated scholars across diverse fields for over 3,000 years, dating back to early Chinese attempts to produce artificial silk [14]. Biomimetics can be defined as the interdisciplinary cooperation between biology, technology, and other fields of innovation, aimed at solving practical problems through the functional analysis of biological systems, the abstraction of these systems into models, and the subsequent transfer and application of these models to technological solutions [15]. A definition of bio-inspiration would be a creative approach based on the observation of biological systems [15]. A set of steps (methodology) for implementing biomimetics includes: (1) problem analysis; (2) abstract technical problem; (3) transpose to biology; (4) identify potential biological models; (5) select biological model of interest; (6) abstract biological strategies; (7) transpose to technology; and (8) implement and test initial model [15]. The topics covered by this field vary, demonstrating the broad impact that biomimetics and bio-inspiration can have across various sectors and branches of society, from science-driven technology and innovation in business to the economy. Recent advancements in this area address, among other things, materials and surface technologies, fluid dynamics, robotics, and architecture [16].

Optimisation problems have also been addressed by this field, including goods transportation, water distribution, resource allocation, operational optimisation, and others. A successful example of nature-inspired solutions is algorithms for solving optimisation problems [17]. Nature-inspired algorithms (bio-inspired algorithms) are among the most powerful algorithms for optimisation and have a significant impact on computing [18]. A study focusing on coordinating and controlling autonomous vehicles, based on learning from collaborative animals and the bio-inspired algorithm (Particle Swarm Optimisation), was conducted. It demonstrated economic feasibility when compared to the conventional alternative. In another study, a swarm of small electric robotic tractors, collectively having the same operational capacity as a large conventional agricultural tractor, was compared [19]. The cost and operational capacity analysis between a 270 kW tractor and a swarm of ten 24 kW robotic tractors showed the following results:

- The robotic swarm has the same field capacity in hectares per hour for ploughing as the conventional tractor (1.4 ha/h).
- The acquisition cost of a conventional tractor (US\$ 355,400.00) is higher than the cost of the robotic swarm (10 x US\$ 25,316.00).
- The operational cost is 3.2 times higher for the conventional tractor per hour compared to the swarm's operational cost for the same field capacity.

- A preliminary design for an electric robotic tractor was proposed, and research challenges were suggested to operationalise the idea [19]. This can be considered a positive result and may be an interesting path for other areas.

There is still much to explore in the topic of biomimetics and bio-inspiration, and for more details, it is recommended to consult the following works [14–16,20–23].

S1-5. Collaborative animals

Collaborations or social interactions in which two or more individuals coordinate their behaviours to produce outcomes from which both benefit are common in nature [24]. Individuals from many species engage in behaviours that benefit others or bring advantages to both the agent and the recipient [24]. Social insects (ants, bees, termites, and others) are an example, as they cooperate and form organised groups of individuals that have evolved into such well-integrated societies that they effectively function as new individuals, superorganisms [25].

Termites (Isoptera), such as social cockroaches, have evolved immensely and highly differentiated societies from a rather simple insect blueprint. They are supreme converters in the soil, from the tropics to deserts; they churn and mix, and with the help of bacteria, protozoa, and fungi, they recycle cellulose fibres [25]. Ants, bees, and wasps (*Hymenoptera*) evolved from more advanced, sensitive, and subtle insects that tolerate and utilise light, ranging from the equator to polar regions. They also started as scavengers and hunters but began to form many mutualistic associations with plants and other insects, acting as protectors, predators, scavengers, honeydew producers, pollinators, and seed dispersers. Their societies can be small or vast, mobile or stationary, with high communication and integration capabilities [25]. The behaviour of these organisms has evolved to balance priorities between various needs, such as growth, survival, and reproduction [26].

Their success is unquestionable; they are ubiquitous and have high ecological activity, adaptation, and resilience [25]. They are constantly managing multiple demands on their time and energy, and decisions related to relevant ecological trade-offs have important consequences for their fitness [27]. From the animal's perspective, the fundamental problems are: what are the best contexts to choose, and which stimuli are important and require a response to achieve a specific goal, such as homeostasis, survival, or reproduction [26]. From a decision analysis perspective, the context to choose corresponds to selecting the best alternative, and defining the important stimuli corresponds to the criteria and their respective weights. Thus, a proper understanding of the decision-making mechanisms in collaborative animals allows for their equivalence with decision-making processes in institutions where individuals coordinate their behaviours to produce mutual benefits. Thus, a study compared the cooperative resolution of a geometric puzzle between ants and humans, and the main findings were: ants demonstrate natural scalability in cooperation due to the simplicity of their individual rules, allowing a large group to act efficiently; humans, on the other hand, need extensive communication to coordinate effectively, and when this communication is restricted, their collective performance worsens [28]. The topic of decision-making by collaborative animals and its

applications can still be explored, and for further information, it is recommended to consult the literature [27,29–34].

S1-6. Two-Alternative Forced Choice (2AFC)

The two-alternative forced choice (2AFC) is a common paradigm used to study the decision-making behaviour of animals [35]. In each test, an animal is presented with a stimulus s_t and it makes a choice $p_R \in \{0, 1\}$, commonly referred to as a "left" or "right" choice. The animal receives a reward ($r_t = 1$) if its response p_R is correct, but no reward ($r_t = 0$) if the response is incorrect. The correct response p_R to a given stimulus s_t depends on the specific task rules. We assume that the behaviour of an animal in a trial t is governed by an internal model, parametrised by a set of weights w_t . These weights describe how the animal's choice depends on the task stimulus (s_t), as well as on any aspects irrelevant to the task, such as a bias independent of the stimulus (w_b) towards one choice over the other [35]. The input vector x_t couples linearly with the weight vector w_t , so that the probability of the animal making the "right" choice in trial t can be expressed as follows:

$$x_t = [1, s_t] \quad (\text{SMP1.3})$$

$$s_t = [s_1, s_2, \dots, s_n] \quad (\text{SMP1.4})$$

$$w_t = [w_b, w_1, w_2, \dots, w_n] \quad (\text{SMP1.5})$$

$$p_R = \frac{\exp(x_t \cdot w_t)}{1 + \exp(x_t \cdot w_t)} \quad (\text{SMP1.6})$$

where: p_R is the probability of the animal making the "right" choice"; $w_1, w_2, \dots, w_n$ is the set of weights.

The weight set that interacts with the constant equal to 1 in the input vector captures the bias towards choosing the "right" option, while the weights that interact with the task stimuli s_t represent the animal's sensitivities to the stimuli. This model is generally applicable to a wide range of tasks and can be easily extended to include other types of covariates for the task, such as history dependence, homeostasis, survival, and reproduction [26,35]. It is also a natural extension of the classical psychometric curve approach to model choice behaviour [35]. The performance of the 2AFC model is measured by the proportion of correct responses, which ranges from a chance level of 0.5 for very weak signals to 1.0 for very strong signals [36]. For more information on the fundamentals of the 2AFC model, it is recommended to consult the literature [34–38].

S1-7. The Herfindahl-Hirschman Index (HHI)

What came to be known as the Herfindahl–Hirschman Index (HHI) was developed by Herfindahl in his dissertation [39,40]. The objective of the dissertation was to determine whether the steel industry had become more or less monopolistic over time. To assess this, Herfindahl required a measure of monopoly power; consequently, the HHI was developed [39,40]. The HHI is calculated using the following formula:

$$HHI = \sum_{i=1}^N S_i^2 \quad (\text{SMP1.7})$$

$$S_i = \frac{x_i}{\sum_{i=1}^N x_i} \quad (\text{SMP1.8})$$

where: N represents the total number of firms in the market; S_i is the market share of the i-th firm; and x_i represents the output share of the i-th firm. HHI values range from 0 (perfect competition, in which the market shares of all firms are close to 0) to 1 (pure monopoly). According to the antitrust guidelines of the United States Department of Justice, a market is considered competitive when $HHI \leq 0.100$, moderately concentrated when $0.100 < HHI \leq 0.180$, and highly concentrated when $HHI > 0.180$ [39,40]. Further information on the HHI can be found in the literature [39,40].

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